

# CAUSAL FERMION SYSTEMS

The theory of causal fermion systems represents a novel approach to fundamental physics and is a promising candidate for a unified physical theory. This book offers a comprehensive overview of the theory, structured in four parts: The first lays the necessary mathematical and physical foundations; the second offers an introduction to the theory and the causal action principle; the third describes the mathematical tools for analyzing causal fermion systems; and the fourth gives an outlook on the key physical applications.

With relevance across mathematical and theoretical physics, the book is aimed at graduate students and researchers interested in novel approaches to the structure of spacetime and alternative perspectives to the more established quantum field theories. It can be used for advanced courses in the subject or as a reference for research and self-guided study. Exercises are included at the end of each chapter to build and develop key concepts.

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# Causal Fermion Systems

An Introduction to Fundamental Structures, Methods and  
Applications

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